La implicación de la memoria de trabajo en la resolución mental de problemas aritméticos: el caso de la discalculia

The involvement of working memory in mental arithmetic problem solving: the case of dyscalculia

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Resumen
Para resolver problemas aritméticos los humanos necesitamos tratar con diferentes piezas de información. Por esta razón, parece plausible que necesitemos de algún sistema capaz de procesar, retener y manipular dicha información. La memoria de trabajo es el sistema encargado de llevar a cabo estos procesos, por lo que podría estar implicado en la resolución de operaciones aritméticas. En este sentido, la investigación empírica ha mostrado que los diferentes componentes de la memoria de trabajo (el ejecutivo central, el bucle fonológico, y la agenda visoespacial) juegan diferentes roles en el proceso de resolución de problemas aritméticos. Por otro lado, algunos investigadores sugieren que la discalculia evolutiva, la dificultad para llevar a cabo operaciones matemáticas más frecuente en la población, se caracteriza por un déficit principal en la memoria de trabajo. Sin embargo, estos resultados no han sido siempre replicados. En este artículo se presenta una revisión actualizada de la implicación de la memoria de trabajo en la resolución de operaciones aritméticas. Como se verá, cada componente cumple con una función específica en el proceso de resolución de operaciones aritméticas. Además, la evidencia en contra y apoyando un déficit principal en memoria de trabajo en pacientes con discalculia del desarrollo será revisada.

Palabras clave: memoria de trabajo, discalculia, ejecutivo central, bucle fonológico, agenda visoespacial, buffer episódico

Abstract
To solve arithmetic problems, humans need to deal with several pieces of information. The working memory system actively stores and manipulates information. For this reason, it seems plausible that this system is involved in solving arithmetic problems. In fact, it has been shown that the different subcomponents of working memory (central executive, phonological loop and visual sketchpad) play different roles in solving arithmetic problems. On the other hand, some research has shown that the most typical arithmetic disability, that is developmental dyscalculia, is characterized by a main deficit in working memory. However, these results have not been always obtained. In the present article the involvement of working memory in solving arithmetic operations is reviewed. Moreover, the evidence supporting and against a main impairment in working memory in people with developmental dyscalculia will be discussed.

Keywords: working memory, dyscalculia, central executive, phonological loop, visual sketchpad, episodic buffer
Introduction

Solving simple mathematical problems is so common that frequently we are not aware of its importance. In fact, without this ability it would be impossible to go shopping, to understand our watch or to know whether our football team won or lost. Fortunately for most of us, we solve these kinds of problems easily. Cognitive psychologists have been trying to understand our mathematical skills for several decades, in spite of this, what processes are involved in solving mathematical problems are the target of an intense debate (Dehaene, 1992; DeStefano & LeFevre, 2004; Heathcote, 1994; Trbovich & LeFevre, 2003; see Campbell, 2005 for a review).

Solving an arithmetic problem involves a variety of mental activities. In this sense, if the operation 37+4 is presented, the steps to solve it may be as follow (Trbovich & LeFevre, 2003): a) accessing a memory representation for the units; b) retaining the intermediate sum; c) increasing the decade; d) assembling an answer; e) generating a phonological code. In order to deal with all this information, we would need to store it temporally and to carry out different processes on it (see DeStefano & LeFevre, 2004). The working memory system seems to be in charge of conducting these functions. Baddeley and Hitch (1974) defined working memory as a limited capacity system which actively maintains and stores information temporarily for its subsequent use.

Baddeley and Hitch (1974; Baddeley, 2000, 2007) proposed a multi-component model of working memory. In its first formulation (Baddeley & Hitch, 1974), the multi-component model was comprised by a central executive, which coordinates the function of the rest of the subcomponents, the phonological loop and the visual-spatial sketchpad, which deal with verbal information and visual-spatial information, respectively. Recently, Baddeley (2000, 2007) added a fourth component: the episodic buffer. According to him, the main function of this component is to integrate information from the rest of the subcomponents and from the long-term memory (Baddeley, 2000; Baddeley & Wilson, 2002). This model has been proposed to explain a great amount of data from different cognitive tasks, such as reading (e.g., Daneman & Carpenter, 1980), vocabulary acquisition (e.g., Gathercole & Baddeley, 1996), etc. Nevertheless, research about the role of working memory in arithmetic is scarce (Butterworth, Cipolotti & Warrington, 1996; DeStefano & LeFevre, 2004; Geary, 1993; Raghubar, Barnes & Hecht, 2010; Trbovich & LeFevre, 2003).

Dyscalculia is an important impairment to conduct arithmetic operations (Butterworth et al., 1996; Butterworth & Yeo, 2004 García-Orza, in press; Geary, 1993; Passolunghi & Siegel, 2011). Its prevalence is between 3.6% and 6.5% (Butterworth & Yeo, 2004), albeit some researchers increased this percentage until 10.9%. This impairment can be considered as a developmental disorder (Developmental dyscalculia; hereafter dyscalculia) or as an acquired disorder as a consequence of brain injury (acalculia or acquired dyscalculia). There are several differences between these disorders, but they have in common an impairment in the arithmetical domain (see Ardila & Rosselli, 2002). However, although in developmental dyscalculia some evidence showed working memory impairments, this evidence is not found in acalculia (see Butterworth et al., 1996). For this reason, here only data about developmental dyscalculia will be discussed. Some researchers have claimed that this disorder is secondary to more general cognitive disabilities. For example, Geary (1993; Geary, Hamson & Hoard, 2000; Mabbot & Bissanz, 2008) established that one of the main problems underlying the developmental dyscalculia is a primary deficit in working memory. However, this working memory involvement has not always been found (Landerl, Bevan, & Butterworth, 2004). For this reason, the relationship between developmental dyscalculia and working memory is not clear. The present review is an attempt to draw this relationship. In the next section, the multi-component model of working memory will be associated with the arithmetical domain. In this section we will present some recent data about the relationship between working memory and arithmetic. Later, the involvement of working memory in dyscalculia will be discussed. At the end, some conclusions will be drawn.

Working memory and mental arithmetic

According to DeStefao and LeFevre (2004) a model of working memory has to be able to explain three main points. Firstly, how the information from our senses enters. Secondly, how information is maintained actively. Thirdly, how this information is manipulated for this system. Baddeley and Hitch (1974; Baddeley, 2000; Baddeley, 2007) proposed the multi-component model of working memory (see figure 1), which has been used as an explanation of the vast majority of research in this field. The multi-component model is composed by four independent but interrelated subsystems: the central executive, the phonological loop, the visual sketchpad and the episodic buffer (see Baddeley, 2007).

Figure 1. Multicomponent Model of working Memory.

The central executive is considered the most important, but it is also the least understood component (Baddeley, 2000, 2003). It is assumed to be a supervisor which distributes attentional resources between the other components. Moreover, it plans the sequence of activities to carry out. The central executive has been involved in several activities in arithmetic solving. DeStefano and LeFevre (2004) indicate that the central execu-

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1 In this article, the words dyscalculia, mathematic disability or arithmetic disability will be used interchangeably.
tive may be involved in keeping track of intermediate operations which have already been done. Moreover, it seems that the central executive plays a role in both simple and multi-digit operations (De Rammelaere, Stuiven, & Vandierendonck, 1999; DeStefano & LeFevre, 2004; Hecht, 2002). For example, Hecht (2002) presented single-digits additions with their results. Participants had to decide whether the result was correct or not (verification task). He found that participants who had to generate either random letters or numbers, performed worse. Similar results have been found in multiplication (De Rammelaere, Stuiven & Vandierendonck, 1999) and when production tasks (to say the result) are used instead of verification (Seitz & Shumann-Hengsteler, 2002).

The phonological loop is, possibly, the most researched subcomponent (Baddeley, 2000). It is specialized in holding verbal information. Some researchers have shown that in arithmetic it is used to maintain intermediate results (Heathcote, 1994). For example, if the operation 17+4 is presented, people start adding 7+4, retaining this intermediate result before finishing the whole addition. Dehaene and Cohen (1995) have hypothesized that verbal codes play a fundamental role in multiplications and sums. In this sense, “disrupting” the phonological loop would impair multiplication and sum solving (Lee & Kang, 2002).

The visual-spatial sketchpad is considered to have similar features to the phonological loop, but with visual and spatial information (Logie, 1995). Hitch (1978) showed that the visual-spatial sketchpad serves to represent operations into the space. In the field of number processing, Dehaene and Cohen (1995) have stated that visual and spatial codes are used in tasks such as subtraction and magnitude comparison.

Specifically, these three components have been shown to be involved in early stages of arithmetic (DeStefano & LeFevre, 2004; Trbovich & LeFevre, 2003; Raghubar et al., 2010), that is in encoding processes. In this sense, Trbovich and LeFevre (2003) investigated the role of presentation format (horizontal vs. vertical) on two plus one-digit mental additions. They hypothesized that when the addition is presented in horizontal format, it is more probable that participants rely on phonological codes, because they would maintain intermediate results in the phonological loop (Heathcote, 1994; Trbovich & LeFevre, 2003; DeStefano & LeFevre, 2004). In this case, a phonological load would disrupt this arithmetic task. On the other hand, when the addition is presented in vertical format, it is more probable that participants rely on visual-spatial codes, because they would use the same unit-to-decade algorithm that they would use in written arithmetic. If it be so, a visual load would have negative consequences in the arithmetic task.

Using visual and verbal load, Trbovich and LeFevre (2003) presented to their participants both vertical and horizontal sums. Participants had to say the result of the operation while they maintained either a visual (a pattern of asterisk) or a phonological (consonant-vowel-consonant pseudoword) load. They found that under phonological load, performance is strongly affected when problems are presented in horizontal and moderately affected when they are presented in vertical. These results suggest that horizontal problems require more phonological than vertical codes. On the other hand, under visual load although it had no effect on either vertical or horizontal positions problems, participants maintained worse the visual load when problems were presented in vertical format. This result seems to indicate that vertical sums do involve visual codes (Trbovich & LeFevre, 2003; DeStefano & LeFevre, 2004).

In summary, Trbovich and LeFevre (2003) showed that both phonological loop and visual-spatial sketchpad are involved in multi-digit additions. On the other hand, Lee and Kang (2002) showed that phonological loop and visual-spatial sketchpads are involved even in simple multiplication and subtraction, respectively (but see De Rammelaere et al. 1999).

Although the format of presentation seems to play an important role in mental addition, it is unknown how the format of presentation affects subtraction solving. Dehaene (1992) established that subtractions are solved using visual-spatial codes. If so, when a subtraction is presented horizontally, because it needs to be recoded in vertical formats, it would need more visual resources than when it is presented in vertical. For this reason a visual load would have worse consequences in horizontal format.

Recently, Baddeley (2000) added a fourth component in his model: the episodic buffer. It is considered to integrate features from the phonological loop, the visual-spatial sketchpad and long-term memory (see Baddeley & Wilson, 2002). In its first formulation it was considered to be controlled by the central executive (Baddeley, 2000, 2003, Repovs & Baddeley, 2006). However, the role of the central executive on the episodic buffer is not clear (Allen, Baddeley, & Hitch, 2006; Baddeley, 2007; Baddeley, Hitch, & Allen, 2009). Although, to my knowledge, there is no direct research about the role of the episodic buffer in number cognition, some evidence suggests that it may be important in a great variety of numerical tasks. For example, a well-documented phenomenon in the number processing field is the size-congruity effect (Henik & Tzelgov, 1982). This effect is shown when two different numbers are presented in different physical sizes. Participants have to decide which number of the pair is numerically larger. This effect shows that participants are faster in judging the numerically larger number if it is congruent with its physical size (Henik & Tzelgov, 1982). This effect implies the bounding of semantic features from long-term memory (numerosity of the numbers displayed) and from short-term memory (physical size). This process may be carried out by the episodic buffer. On the other hand, when people have to solve an operation, they can transform it to make it easier (9+6 = 10+5, see McLean and Hitch, 1999). In order to make this transformation, people have to integrate information from the current information (i.e. 9+6) and from long-term memory (10+5). Possibly this function is also carried out by the episodic buffer. Additional research is needed to elucidate whether the episodic buffer is in fact involved in these processes.

In conclusion, phonological loop, visual-spatial sketchpad and central executive play an important role in arithmetic. The role of the episodic buffer has not been researched yet, although some indirect evidence seems to indicate that it is also important in some arithmetical task. In this sense, it is necessary to specify its function, if it has, in arithmetic.

**Working memory impairments in dyscalculia**

According to Geary (1993), children with dyscalculia use the same strategies as normal children, such as verbal counting. However, they fail in the accuracy, strategy mix, and in the pattern of developmental change (Geary, Hoard, Byrd-Craven,
& DeSoto, 2004). Counting processes allow children to maintain the operation actively in their memory (Geary et al., 2000). If the operation is overlearned, it will be stored in the long-term memory (Siegel & Shrager, 1984; Geary, 1993). So, the more times one operation is solved, the better it will be recollected. In this sense, it seems that children with dyscalculia, due to their problems in working memory, cannot transfer the information to long-term memory (see Geary et al. 2004).

Some empirical evidence has shown that working memory is impaired in children with dyscalculia. However, what aspects are impaired remains under debate. Hitch and McAuley (1991) found phonological deficits but only in numerical working memory task. In a similar way, Siegel and Ryan (1989) found that children with dyscalculia showed a similar performance in a verbal working memory to control-matched. However, their performance in numerical working memory was impaired. These results seem to indicate that children with arithmetical disability show a working memory deficit with numerical information.

Nevertheless, not all studies support this assumption. In this sense, Swanson (1993) found a general working memory deficit in both children and adults with arithmetical disability. More recently, Passolunghi and Siegel (2001) examined children performance in a great amount of working memory tasks. They matched groups in verbal intelligence and reading ability to rule out alternative explanations. They showed that children with dyscalculia showed a similar performance in working memory tasks involving verbal and numerical information. These results show evidence for a general working memory deficit in dyscalculia. Interestingly, Passolunghi and Siegel (2001) demonstrated that children with dyscalculia do not present deficits in short-term memory task. In this sense, it seems that the problem of these children consist in the maintaining and refreshing of information in their memory, rather than in passive memory. More recently, Landerl et al. (2004) found that children with dyscalculia were normal or above average on a great variety of tasks involving phonological working memory, non-verbal intelligence, non-numerical verbal information, language and psychomotor skills. According to them, dyscalculia is better defined as a specific deficit in the processing of numerical information.

In summary, it is not clear whether children with arithmetical impairments show a general impairment in phonological working memory or not. Some studies show a phonological deficit but only with numerical information (Siegel & Ryan, 1989; Hitch & McAuley, 1991), whereas others found a general phonological working memory deficit (Swanson, 1994; Passolunghi and Siegel, 2001). Even others did not find phonological working memory deficits (Landerl et al. 2004). Disagreement in these data could be due to problem with control subjects. In this sense, it is necessary to control some variables such as intelligence and reading abilities to obtain a faithful assessment (see also Landerl, et al. 2004).

Others researchers have explored the role of visual-spatial working memory in dyscalculia. For example, McLead and Hitch (1999) found spatiotemporal working memory deficits in children with poor arithmetic skills. According to them, this deficit shows that spatiotemporal working memory is essential to cope with numerical information. More recently, Schuchardt, Maehler and Hasselhorn (2008) reported similar results. It is worth noting that they controlled IQ and carried out a rigorous selection of their participants. In this sense, they formed two groups: children with a specific disability in mathematics and children with dyslexia but moreover with problems in arithmetic. Schuchardt et al. (2008) found impairments in the visual-spatial sketchpad in the first group. In the dyslexic with mathematics disabilities group they found phonological impairments but not visual-spatial. However, others researches did not find any relationship between visual-spatial sketchpad and mathematical disability (Geary et al. 2004; Bull, Johnston & Roy, 1999). As in the phonological loop case, it is not clear whether the visual-spatial sketchpad is impaired in children with dyscalculia. Again, it seems that some non-controlled variables may be influencing in these contradictory results.

Regarding the central executive, most researchers agree that it is impaired in dyscalculic children (see Schuchardt et al. 2008). For example, Passolunghi and Siegel (2001) showed evidence suggesting that people with arithmetic impairments have an inability to ignore irrelevant information. In this sense, when these researchers presented to their participants a list of number to recall, they mixed numbers from the current and previous trials. According to Passolunghi and Siegel (2001), these intrusions errors showed that, although dyscalculic children maintain information in working memory, they are not able to suppress it in subsequent trials. Instead of the general agreement in the role of central executive in children with dyscalculia, it is worth noting that Landerl et al. (2004) did not find such impairment.

Discussion

It seems clear that working memory plays an important role in arithmetic and calculus (see DeStefano & LeFevre, 2004). The multi-component model of working memory (Baddeley & Hitch, 1974; Baddeley, 2000; Baddeley, 2007) has been proposed to explain how we cope with arithmetic operations (Trbovich & LeFevre, 2003; DeStefano & LeFevre, 2004). In this sense, the central executive has been involved in keeping track the part of the operation which has been done (DeStefano & LeFevre, 2004). Secondly, the phonological loop plays an important role in maintaining intermediate results (Heathcote, 1994). Thirdly, the visual-spatial sketchpad is used to represent operation into the space (Hitch, 1978). Lastly, although there is no direct evidence of the involvement of the episodic buffer in arithmetic, as stated before, it can be implicated, for example, in the size congruity effect (Henik & Tzelgov, 1982) or in the transformation of operation in simpler numbers (e.g. 9+6 = 10+5; McLean and Hitch, 1999).

Research has been conducted in order to specify the role of working memory in developmental dyscalculia. In general terms, some researchers have found a general working memory deficit in children with dyscalculia (Swanson, 1993; Passolunghi and Siegel, 2001), whereas others (Siegel and Ryan, 1989; McLean and Hitch, 1999) have found such deficit but only with numerical information. On the other hand, phonological loop has sometimes been involved as the central deficit in dyscalculia in some studies (Hitch & McAuley, 1991 Swanson, 1994; Passolunghi & Siegel, 2001), whereas others have involved the visual-spatial sketchpad (McLean & Hitch, 1999; Schuchardt et al. 2008). Even others have not found relationships between dyscalculia and working memory (Landerl et al. 2004). However, it is worth noting that when working memory impairment has been found, they are well explained by the multi-component
model of working memory (Baddeley & Hitch, 1974; Baddeley, 2000; Baddeley, 2007).

As can be observed, research findings about the role of working memory in dyscalculia are quite heterogeneous (Schuchardt et al. 2008). Probably, the reasons for these disagree-ments are the variability of the sample used. In this sense, some researchers have found that 64% of children with dyscalculia had also dyslexia (Lewis, Hitch, & Walker, 1994). Most studies reviewed examined dyscalculia without taking into consideration this comorbidity, so it seems reasonable that many differ-ences between these studies may be explained by a non-controlling sample. Future research should take into consideration this fact, trying to control the different impairments in the sample. In spite of the fact that the cognitive aspects of dyslexia and dyscalculia are considered to be different (Butterworth, 2010; Ramus, 2003), it is not an easy task to find the two dis-abilities dissociated (Badian, 1999; Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2011; García-Orza, in press). For example, Compton et al. (2011) found that 10.1% of their participants had problems in calculus, 6.6% had problems in reading, and almost 4% had comorbidity. This problem of comorbidity makes it difficult to extract conclusions about both impairments. For this reason, future research should also determine what cogni-tive aspects are shared and exclusive of both dyslexia and dyscalculia.

References


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